

Milli-kelvin Thermodynamic and Transport Measurements of Low Dimensional Systems in High Magnetic Fields

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.....Martin J Smith

Abstract

This thesis presents an investigation into aspects of the integer quantum Hall effect, specifically the near-dissipationless state of the longitudinal resistivity ρ_{xx} between Landau levels, and the associated broadening of the levels. Eddy currents induced by a time varying magnetic field B are considered in chapter 4. The temperature dependences of the eddy currents were measured over the range 100 mK to 1600 mK. The peak current at filling factor $\nu = 2$ was shown to saturate at $\gtrsim 800$ mK, more robust than previously observed, but was reduced by elevating the temperature to 1600 mK. The saturated regime is associated with a breakdown of the quantum Hall effect, and in this case, the most likely candidate for the saturation is an electron heating effect.

Sweep-rate dependences were characterised for a range of filling factors and temperatures, and even for the lowest sweep rates, never entered a linear regime. Induced currents $\nu = 1, 2$ and 4 all saturated at the same critical value at 100 mK, but $\nu = 4$ was shown to reduce with slower sweep rates, consistent with the prediction that the ρ_{xx} minima is not as small as for lower Landau levels. Induced current decays were measured to be similar to previous work, a fast initial decay attributed to breakdown of the QHE followed by a much longer slow decay. The eddy decay of $\nu = 1$ at low temperature, in the slow decay regime, is among the most persistent reported. It was shown that the assumptions of previous work had not evaluated the mutual inductance of the eddy current in the presence of the magnet sufficiently. By fitting a suitable function to the IV characteristic of $\nu = 1$ the shape of the induced current was modeled. The model agreed with the data, producing a similar shape and a very long time constant for the slow decay.

In chapter 5 the hysteresis in the magnetoresistance of a quantum point contact was investigated, through a simultaneous transport and magnetometry measurement. Induced currents corresponding to filling factors up to $\nu = 8$ were measured. Three corresponding features were measured in the magnetoresistance of a QPC, one more than previously seen. The temperature dependence was measured simultaneously, and for Landau level filling factor $\nu = 1$, the general shape of the curves was the same. The sweep rate IV characteristics of the the two experiments were similar. Sweeping the magnetic field B to a fixed field position and waiting, demonstrated that both phenomena decay with time, a fast decay of seconds and a slow decay taking more than 10,000 seconds. An attempt was made to affect the induced eddy current by switching the QPC gate on/off. Experiments on a fast timescale, 10 ms, resolved structure in the induced currents that has previously been attributed to the noisy breakdown of the quantum Hall effect. By performing a simultaneous measurement, individual breakdown events were seen and correlated.

After investigating the zero-resistance state in chapter 4 and chapter 5 with induced currents, exactly how the zero-resistance state varied between Landau levels was the topic of chapter 6. A method was presented for the fabrication of a novel device, to measure the magnetisation and the heat capacity of a 2DES at the same time. AuGe thin film resistors were grown in only 10 bilayers, reducing the heat capacity per unit area by approximately an order of magnitude on previous workers. The AuGe thermometers were shown to be ‘tunable’, i.e. the temperature dependence was dictated by the annealing conditions after growth, so thermometers with different gold concentrations due to growth conditions, could be tuned to have similar temperature dependences. Low temperature thermometers with small heat capacities were repeatably produced, and thermometer D5 is presented in this thesis with a sensitivity of $S = 0.58$.

At an elevated refrigerator temperature of nearly 300 mK, heat pulses of ~ 26 nJ were resolved on a device which had a 100% front processing success rate, but was not etched from the back. It was shown that a device to measure the broadening of the low temperature, high magnetic field 2DES density of states is possible.

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Contents

1	Introduction	21
2	Background	24
2.1	Two-dimensional electron systems	24
2.1.1	Construction of a Two Dimensional System	24
2.1.2	Two Dimensional Density of States	28
2.1.3	Two Dimensional Electrons in a Magnetic Field	30
2.2	Oscillating Magnetic Moment - the de-Haas—van Alphen effect . . .	34
2.3	Oscillating Heat Capacity	37
2.4	The form of the density of states	40
2.5	The Hall effect	45
2.6	The integer quantum hall effect	47
2.7	Edge States	48
3	Experimental Details	51
3.1	Introduction	51

CONTENTS

3.2	The Dilution Refrigerators	51
3.2.1	Oxford Instruments Refrigerator System	51
3.2.2	Cryogenic Consultants Limited dilution refrigerator	52
3.3	Vibration Damping	53
3.4	Magnetometry	54
3.5	Calorimetry	55
3.5.1	Adiabatic Calorimetry	56
3.5.2	Relaxation Calorimetry	58
3.5.3	AC Calorimetry	59
3.6	The Samples	60
4	Measurements of induced currents in the QHE	63
4.1	Introduction	63
4.2	Early measurements of induced currents	64
4.3	High current breakdown of the quantum Hall effect	65
4.3.1	Electron-heating model	65
4.3.2	Intra-Landau-level scattering	67
4.3.3	Inter-Landau-level scattering	67
4.4	Experiment	69
4.5	Temperature Dependence	74
4.6	Sweep rate IV curves	82

CONTENTS

4.7	Decay measurements	88
4.7.1	Brief Introduction	88
4.7.2	Establishing a background	89
4.7.3	Decay of filling factors $\nu = 1, 2$, and 4	90
4.8	Relating IV characteristics to induced current decays	94
4.8.1	Energy stored in an induced current	95
4.8.2	Discharging an eddy current	100
4.9	Conclusions	104
5	Hysteresis in the conductance of a QPC	106
5.1	Introduction	106
5.2	Experimental setup	111
5.3	Characterising QPC AK-47	112
5.3.1	Magnetoconductance	118
5.4	Temperature dependence	122
5.5	Sweep rate dependence	126
5.6	Decay measurements	128
5.7	Correlation of structure in ‘noisy’ breakdown of the QHE	129
5.8	Affect of the gate on the induced current	132
5.9	Conclusions	133

CONTENTS

6	Dual Calorimetry/Magnetisation Measurements of 2DES	135
6.1	Introduction	135
6.2	Cantilever viability	136
6.2.1	Previous measurements	136
6.2.2	Experiment details	137
6.2.3	Preliminary Cantilever data	140
6.3	Calorimetry	142
6.3.1	The Dual Cantilever-Calorimeter	143
6.3.2	Heaters and tracks	146
6.3.3	Thermometry	149
6.3.3.1	Temperature Dependence and Sensitivity	155
6.3.3.2	Magnetic field dependence	158
6.3.3.3	Noise	159
6.3.4	Processing of Device	160
6.3.5	Contacting the Calorimeter and external connection	166
6.3.6	Preliminary Calorimetry Measurements	167
6.4	Conclusions	169
7	Conclusions and Further work	171
7.1	Conclusions	171
7.2	Further work	175
	Bibliography	177

List of Figures

2.1	A simplified heterojunction: two separate n- and p-type semiconductors with different bandgaps (a), are brought together to form a heterojunction (b), confining a 2DES at the electrostatic boundary.	25
2.2	Illustration of the ground state wavefunction of a heterojunction.	26
2.3	The density of states for a 3D and 2D system. The position of the Fermi energy is indicated for the condition that only the first subband is occupied at a finite temperature.	30
2.4	The theoretical density of states for a 2DES in a magnetic field (left), and broadened levels due to disorder and temperature, consisting of both localised and extended states (right).	32
2.5	Oscillation in the Fermi energy due to Landau level quantisation.	34
2.6	Oscillations in the Free energy due to a varying magnetic field B	35
2.7	Oscillations in the magnetisation of a 2DES in a magnetic field, the de-Haas—van-Alphen effect.	37
2.8	Oscillations in the Fermi energy give rise to an oscillating heat capacity, calculated by Zawadzki and Lassnig [24]. There are two contributions, an intra-LL contribution, and an additional inter-LL contribution in the form of spikes.	39

LIST OF FIGURES

2.9	DOS $g(E, B)$ calculated for a 2DES in GaAs for Lorentzian broadened LLs (solid curve) and Gaussian broadened LLs (dashed curve), after Potts et al. [25].	42
2.10	Hall bar geometry of a 2DES. A magnetic field B is applied perpendicular to the plane and the direction of current flow. The voltmeters indicate the longitudinal and Hall voltages.	45
2.11	Experimental data of the quantum Hall effect, characterised by deep resistance minimum in the longitudinal voltage drop, and quantised plateaux in the Hall voltage, from Cage [33].	47
2.12	Edge channels form in a disordered potential. The energy of the edge state increases as the edge of the sample is approached. As the Fermi energy is increased, more Landau levels contribute to the conduction. In this example, the $n = 3$ level is partially full and there are states occupied in the bulk. Image after Beenakker and van Houten [40]. . .	50
3.1	Torsion balance magnetometer. A magnetic field B causes a magnetic moment m in the sample (quantum point contact AK47 pictured without wires), which results in a torque that can be measured by a differential capacitive change. The sample is not to scale.	54
3.2	Heat-pulse calorimetry: A heat pulse (a) results in a thermometer response: (b) Measured response of a AuGe thin film resistor connected to a weak thermal link and (c) Predicted response in quasi-adiabatic conditions.	56
3.3	Ac calorimetry method. The measured heating is at twice the frequency of the applied heater voltage.	59

LIST OF FIGURES

4.1	Current-voltage characteristic of a GaAs/(Al,Ga)As heterostructure at $T = 1.4\text{ K}$, from Ebert et al. [54]. The top inset indicates the magnetic field position at which the breakdown was measured, and the bottom inset shows the device geometry. The solid line in the central figure is magnified by a factor of 50 000 to give the broken curve.	66
4.2	Quasi-elastic inter-Landau-level scattering representation, after Eaves and Sheard [59] Landau levels are sloped ($slope = eE$) due to an electric field. States in the lower LL (ℓ) acquire the same energy as the upper LL ($\ell + 1$) and can tunnel to these new states if perturbed.	69
4.3	Measurement of magnetisation through torque magnetometry and longitudinal resistivity ρ_{xx} through a transport measurement. The resistivity ρ_{xx} is characterised by deep resistance minimum at integer filling factors. Both experiments were conducted at 40 mK but are not simultaneous.	70
4.4	x - and y -phase components of a lock-in amplifier for a typical magnetometry run at a sweep rate of 1.605 mT s^{-1} at 300 mK.	73
4.5	x - and y -phase components taken from figure 4.4 and magnified. Features in the y -phase are small compared to the x -phase, and likely to have arisen from capacitive coupling to the 2DES, oscillations are seen with the Fermi energy is between Landau levels. The double peak structure in the y -phase is predicted by Morris [64].	74
4.6	Phase diagram of the breakdown current in the QHE with temperature after Rigal et al. [66]	75
4.7	Induced eddy currents at $\nu = 1, 2, 3, 4$ changed shape and amplitude with temperature. As temperature increases the integral of the total current decreased, but for $\nu = 2$ the amplitude is approximately constant.	77

LIST OF FIGURES

4.8	Eddy current amplitude vs temperature converted from figure 4.7. Additionally data from a high T sweep at 1.6 K is included, the only remnant eddy current was $\nu = 2$. $\nu = 1, 2$ and perhaps 4, saturate at low T due to a breakdown effect, the strongest candidate is electron heating.	78
4.9	Disorder broadened LLs, showing the position of the Fermi energy at integer ν . When a magnetic field is swept, electrons accumulate in, or deplete from, different regions within the 2DES, resulting in regions with quasi-Fermi energies. After Usher and Elliott [69].	81
4.10	Sweep rate IV curves for $\nu = 1, 2, 4$ and 6 at various temperatures are non-linear. Sweep rates have been converted to an EMF and magnetic moments to a current. At higher EMFs the breakdown of the QHE is evident, the current does not exceed a maximum value. . .	84
4.11	Calculated potential profile across a $20\ \mu\text{m}$ Hall bar, as in equation (4.6) after Balaban et al. [73].	86
4.12	Establishing the ‘zero’ of the decay for $\nu = 1$ by mapping the background with an up and down sweep (black), and sweeping out after ~ 10 hours. The current decayed by less than 10% in this time, with most of the dissipation happening in the first 20 s due to breakdown of the zero resistance state.	90
4.13	Induced eddy current decay for filling factor $\nu = 1$. Of the dissipated current, most is lost in the initial part of the decay, followed by an extremely persistent slow decay.	92
4.14	Induced eddy current decay for filling factor $\nu = 2$. A significant portion of the current is dissipated in the initial part of the decay, followed by a persistent slow decay of hours. The step in the 300 mK curve is due to ice cracking on the cryostat, causing a mechanical knock. . .	93

LIST OF FIGURES

4.15	Induced eddy current decay for filling factor $\nu = 4$. Most of the current is dissipated in the initial part of the decay, followed by a slower decay of several hours.	94
4.16	Simplified schematic of the magnet with inductance L_m coupling to the eddy current with an inductance L_e	96
4.17	Calculating dissipation from a sweep rate IV curve for an induced current. Starting at an arbitrary current, a resistance is calculated, and I is dissipated through the resistance R_1 in a time interval dt . A new current is then used calculate a new resistance R_2 , and the process is repeated.	101
4.18	Sweep rate curve for induces current $\nu = 1$ at 800mK, with fitted function.	103
4.19	Decay of $\nu = 1$ at 800 mK. The blue curve is computed from the sweep rate IV curve, and follows the trend of the data.	104
5.1	The first experimental realisation of quantised conductance in zero magnetic field. The constriction is tunable; the width decreases, pinching off conducting channels, as the gate voltage is made more negative. (a) Quantum point contact resistance as a function of gate voltage at 0.6 K. The inset shows the split gate electrodes deposited remotely from the 2DES. (b) Quantum point contact conductance as a function of gate voltage obtained from the data in (a), after an adjustment for a series lead resistance. The conductance shows plateaus at multiples of $e^2/\pi\hbar$. Images taken from Van Wees et al. [79]. . . .	107

LIST OF FIGURES

5.2	Magnetoconductance measurements on a quantum point contact at a sweep rate of $17 \mu\text{T s}^{-1}$ and a temperature of 30 mK, taken from Pioro-Ladrière et al. [81]. (a) The conductance G_{QPC} as a function of magnetic field B , with a hysteretic feature at 3.6 T, corresponding to filling factor $\nu = 2$ (in the 2DES leads). The inset shows an electron micrograph of the device, the scale bar is 300 nm. The quantum dot in the upper gate plays no role in this experiment. (b) (left) An enlargement of the hysteretic feature at $\nu = 2$ and (right) the hysteretic feature at $\nu = 1$	109
5.3	A schematic illustration (not to scale) of the induced current in the 2DES, leads have a Hall voltage which perturbs the potential near the QPC. The radial Hall fields results in a positive charge build up near the gates. The barrier appears shorter to the ballistic electrons and therefore the device has a corresponding dip in magnetoconductance.	110
5.4	Stycast rotor with QPC sample AK47 mounted before being fitted in the magnetometer frame. Insulated copper wires connect to the gold pads on the QPC with silver paint contacts, and are fixed to the edge of the rotor with superglue.	112
5.5	Experimental set-up for measurement of voltage drop along QPC using the primary lock-in as a constant current source. The second lock-in is used to measure the small voltage drop across a $10 \text{ k}\Omega$ resistor, and hence the QPC current, it is removed for the simultaneous experiments.	113

LIST OF FIGURES

5.6	(top) QPC resistance measured at 50 mK with a 10 nA excitation current, and (bottom) The corresponding conductance trace. The conductance can be reduced to below $2e^2/h$ where all conduction is due to tunneling. The sensitivity of the lock-in was set to resolve any steps present, and therefore limits the scale. Only one discrete step in the conductance is seen, at ~ -0.12 V, which corresponds to the number density underneath the gate becoming zero, and the constriction becoming the only remaining current path. The expected steps are smeared, but the gate can still be ‘pinched-off’.	114
5.7	SEM image of AK47. The lithographed width is measured to be 509.4 nm.	115
5.8	Sweeping gate voltage at various fixed magnetic fields corresponding to filling factors. As gate voltage is decreased edge state conducting channels are pinched off and back-scattered at the interface resulting in a resistance plateau. Steps are unexpectedly preceded by an apparent resonance in resistance, strongest for even filling factor transitions.	117
5.9	Cartoon depicting conducting edge channels (red lines) in the 2DES leads. At a fixed magnetic field, corresponding to $\nu = 8$, the number density under the gate is reduced by application of a negative bias voltage. When the number density at the same field corresponds to $\nu = 6$ there are three edge states left and the remaining state in the leads is strongly back-scattered but can possibly still pass through the gate. As soon as n_s is reduced another conducting channel will be pinched off under the gate.	118
5.10	Dual measurement of magnetisation and magnetoresistance	119
5.11	Magnetoconductance of the quantum point contact, obtained from figure 5.10, where $\sigma_0 = 2e^2/h$.	120

LIST OF FIGURES

5.12	Qualitative illustration of the reduction of back-scattering by a magnetic field, responsible for the positive magnetoconductance in the low-field regime in figure 5.11. The electron trajectories approach the constriction without a barrier in a weak magnetic field (left) and a strong magnetic field (right). Image after H. van Houten et al. in ref [40].	121
5.13	Observation of elevating the temperature on the shape and size of the induced current and the hysteretic magnetoresistance with a changing magnetic field at a sweep rate of 62 mT s^{-1} . Features at $\nu = 1$ are suppressed as temperature is increased, both experiments follow a similar trend.	123
5.14	Temperature dependences of induced eddy currents at $\nu = 1$ (top) and $\nu = 2$ (bottom), with corresponding hysteretic magnetoresistances. $\nu = 1$ shows a very similar trend for both experiments, but the QPC intercepts the zero at a lower temperature. At $\nu = 2$ the QPC data suffered from noise, possibly due to trapping charges under or near the gates in the QPCs recent history, however it is clear the magnetisation shows no temperature dependence.	125
5.15	Sweep rate dependences of $\nu = 2$ (top), $\nu = 4$ (bottom) at 100 mK measured simultaneously. It was not possible to reduce the sweep rate enough to leave the saturated regime for $\nu = 2$ due to experimental constraints. The eddy current size is varied and the magnetoresistance was found to mirror the size of the induced current.	127
5.16	Simultaneous measurement of an induced current decay, and the decay of the magnetoresistance for $\nu = 2$ at 300 mK.	128

LIST OF FIGURES

5.17	$\nu = 2$ at 100 mK is observed to have a noise structure similar to that seen in Elliott et al. [76]. The noise is attributed to the breakdown of the QHE; three individual breakdown events are correlated in the simultaneous measurements, magnetisation (black, top) and magnetoresistance (red, bottom).	130
5.18	Induced eddy current at $\nu = 4$ at 300 mK, and a sweep rate of $21 \mu\text{T s}^{-1}$, for two different states: the gate OFF and the gate ON (sample divided into two).	132
6.1	Cantilever T621 to scale. The GaAs cantilever is etched to thickness of $10 \mu\text{m}$	138
6.2	Mounting arrangement for cantilever T621 in the mixing chamber of $^3\text{He}/^4\text{He}$ dilution refrigerator. The back of the cantilever is covered with a thin layer of gold; this and the phosphor-bronze pillar form a capacitor.	139
6.3	Magnetisation data for cantilever T621 at 75 mK, at a magnetic field sweep rate of 3.33 mT s^{-1} . Features correspond to Landau level filling factors, and probably arise from capacitive coupling to the 2DES as they are non-reversing. Hysteresis is seen at $\nu = 1$ and $\nu = 2$, attributed to induced eddy currents.	141
6.4	The Exeter cantilever-calorimeter. The device has two identical cantilevers (only one is shown to have wires for clarity), except that one has a 2DES mesa, and the other has the 2DES removed. All tracks on the cantilever are $50 \mu\text{m}$ wide, except heaters on the 2DES ($62.5 \mu\text{m}$ wide), and 5 nm to 50 nm thick. Tracks outside the dotted red region are gold contact leads $\sim 200 \text{ nm}$ thick.	144

LIST OF FIGURES

6.5	The Exeter cantilever-calorimeter. An enlarged version of the 2DES cantilever from figure 6.4. Heater tracks on the mesa are $62.5\text{ }\mu\text{m}$ wide, all other tracks are $50\text{ }\mu\text{m}$. The guard heater and thermometer create an artificial thermal isolation for the 2DES in a heat capacity measurement.	145
6.6	Example of dirt on the surface causing a break in a thick gold track. .	148
6.7	Thin films of gold for various thicknesses as a function of temperature.	148
6.8	Dependence of room temperature resistivity on annealing temperature, taken from Fortune et al. [94]	151
6.9	Fractal crystallization process as a AuGe film is annealed at 120°C at ~ 10 min intervals in (a)-(g), then (h) an additional 10 min at 135°C . Light contrast is Ge, dark contrast Au. Taken from Zhang et al. [97].	152
6.10	Room temperature resistivity dependence for various anneal temperatures, for resistors of 5 layers and 10 layers.	154
6.11	A log-log plot of the temperature dependence of thermometers B3 and B4, Au:Ge $11.1\text{ }\text{\AA}$: $43\text{ }\text{\AA}$ bilayers after different 30 minute anneals.	155
6.12	Temperature dependence of thermometer D5.	156
6.13	Magnetic field dependence of resistance of thin film thermometer D5.	158
6.14	Zero magnetic field AuGe thin film thermometer D5 noise characteristic, with no driving current.	160
6.15	Orientation of calorimeter and test thermometers in the evaporator. Room temperature resistances are shown to illustrate the deposition gradient of the thermometers.	163

LIST OF FIGURES

6.16	Repair of thin gold tracks. The top image is a magnified version of the bottom. Hot gold damaged the photoresist during evaporation and shorted some tracks. The top image shows a FIB repair, a small trench of material is removed to insulate the tracks (red arrow). Repair carried out by Geoff Hill.	165
6.17	Corner of heater tracks on a cantilever. The etch has gone through the stop and damaged the cantilever.	166
6.18	Calorimeter A4182, contacted with silver paint contacts.	167
6.19	The experimental setup for the preliminary calorimeter experiment. .	168
6.20	Response of thermometer on thinned calorimeter A4182 to constant thermometer heating at 270 mK.	169

List of Tables

3.1	Layer profile of δ -doped heterojunction, sample AK47.	60
3.2	Layer profile of modulation doped heterojunction, sample T621. . . .	61
3.3	Layer profile of modulation doped heterojunction, sample A3970. . .	61
4.1	Critical current densities calculated for different assumptions in the current distribution.	87
4.2	Comparison of energy storage for eddy current at $\nu = 4$, for a satu- rating current of 0.29 mA at base temperature.	99